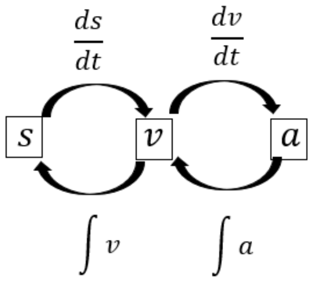
Mathematics Methods

Unit 3

Rectilinear motion

1. Rectilinear motion

Summary:

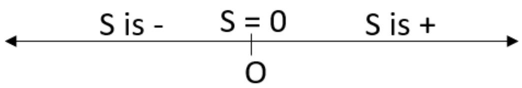


Where,
s: displacement
v: velocity
a: acceleration

(a) Displacement

Displacement (S) of a particle: The distance measured from a fixed point, O, in a specific direction

In layman's term, displacement is how far the particle is from the starting point.



*Displacement is represent by S.

Formula:

$$S = \int_b^a v \ dt$$

Example 1:
The velocity of a moving particle is equated by $v = 2t^3 + 3t^2 - 2t$. Find the displacement of particle after 5 s.

$$S = \int_0^5 v \ dt$$
$$= \int_0^5 2t^3 + 3t^2 - 2t \ dt$$
$$= [\frac{2t^4}{4} + \frac{3t^3}{3} - \frac{2t^2}{2}]_0^5$$
$$= [\frac{t^4}{2} + t^3 - t^2]_0^5$$
$$= 412.5 \ m$$

Example 2:
A particle accelerates at 5 ms^{-2} which is constant along the s-axis. It has a velocity of $-1\ ms^{-2}$ at the start of the journey. Find the displacement of the particle within the range of time, $2 \leqslant t \leqslant 4$.

$$v(t) = -1 + 5t$$
$$S = \int_2^4 v\ dt$$
$$= \int_2^4 -1 + 5t\ dt$$
$$= [-t + \frac{5t^2}{2}]_2^4$$
$$= 28\ m$$

(b) Distance

Distance travelled by a particle: Total length of the path travelled by a particle from its original position to another position.

In layman's term, distance travelled is how far the particle has travelled.

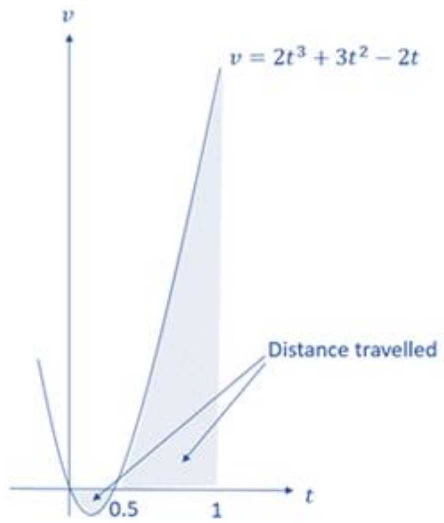
Formula:

$$S = \int_b^a |v|\ dt$$

Tips:
Sketch the graph of velocity function.

Example 1:
The velocity of a moving particle is equated by $v = 2t^3 + 3t^2 - 2t$. Find the total distance travelled by particle from its origin point till $t = 1s$.

$$S = \int_0^1 |v|\ dt$$
$$= \int_0^{0.5} |2t^3 + 3t^2 - 2t|\ dt + \int_{0.5}^1 2t^3 + 3t^2 - 2t\ dt$$
$$= 0.09375 + 0.59375$$
$$= 0.6875\ m$$



Example 2:
A particle accelerates at 5 ms^{-2} which is constant along the s-axis. It has a velocity of $-1\ ms^{-2}$ at the start of the journey. Find the distance travelled by the particle from the origin to when $t = 2\ s$.

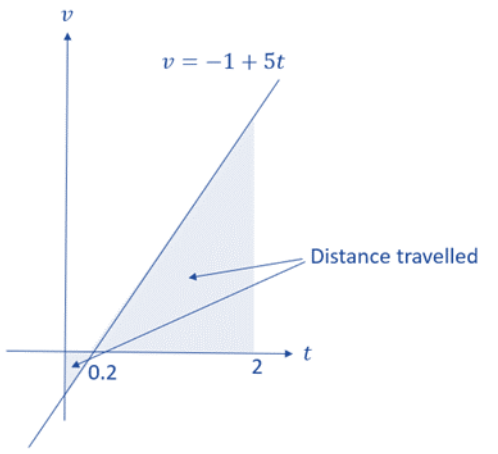
$v(t) = -1 + 5t$

$S = \int_0^2 |v|\ dt$

$= \int_0^{0.2} |-1 + 5t|\ dt + \int_{0.2}^{2} -1 + 5t\ dt$

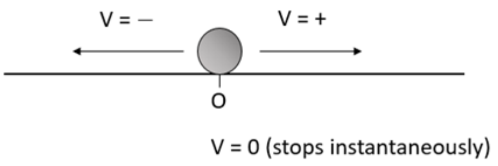
$= 0.1 + 8.1$

$= 8.2\ m$



(c) Velocity

Velocity (v) is the rate of change of displacement.



Formula:

$$v = \frac{ds}{dt}$$

$$v = \int a\ dt$$

Example 1:
The velocity function of a moving particle is given by $v(t) = e^{\sin 2t}$, find the velocity of the particle at $t = 2\ s$. Do not evaluate your answer.

$v(t) = e^{\sin 2t}$

$v(2) = e^{\sin 2(2)}$

$\quad = e^{\sin 4}\ ms^{-1}$

Example 2:

A moving particle travells along a straight line, passing through a fixed point at velocity 12 ms^{-1}. Its acceleration, $a\ ms^{-2}$ is given by $a = 4t^5 + 9$ where t is the time in seconds after passing through O. Find the velocity of the particle when $t = 5$.

$$v = \int (4t^5 + 9)dt$$
$$= \frac{4t^6}{6} + 9t + c$$

The particle passes through O with velocity 12 ms^{-1},

$$12 = \frac{4(0)^6}{6} + 9(0) + c$$
$$c = 12$$

$t = 5$,

$$v = \frac{4t^6}{6} + 9t + 12$$
$$= \frac{4(5)^6}{6} + 9(5) + 12$$
$$= 10473.67\ ms^{-1}$$

(d) Acceleration

Acceleration is defined as the rate of change of velocity.

Formula:

$$a = \frac{dv}{dt} \quad \text{or} \quad a = \frac{d^2s}{dt^2}$$

Example 1:

A moving particle travells along a straight path and passes through a fixed point. Its velocity, $v\ ms^{-1}$ is given by $v = 3t^2 + 6t$. Determine the acceleration of the particle when $t = 2$.

$$v = 3t^2 + 6t$$
$$a = \frac{dv}{dt}$$

$$a(t) = 6t + 6$$
$$a(2) = 6(2) + 6$$
$$= 18\ ms^{-2}$$

Example 2:

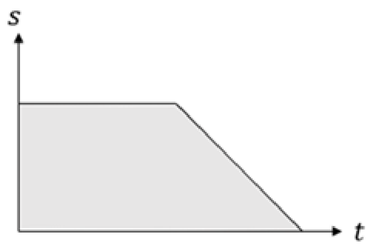
The velocity of a moving particle can be equated by $v(t) = cos\left(\frac{\pi t}{3}\right)$. Find the acceleration of the particle when $t = 3\ s$.

$$v(t) = cos\left(\frac{\pi t}{3}\right)$$
$$v'(t) = \frac{-\pi\ sin\left(\frac{\pi t}{3}\right)}{3}$$

$$a(t) = \frac{-\pi \sin\left(\frac{\pi t}{3}\right)}{3}$$
$$a(3) = \frac{-\pi}{3}\sin(\pi)$$
$$= 0\ ms^{-2}$$

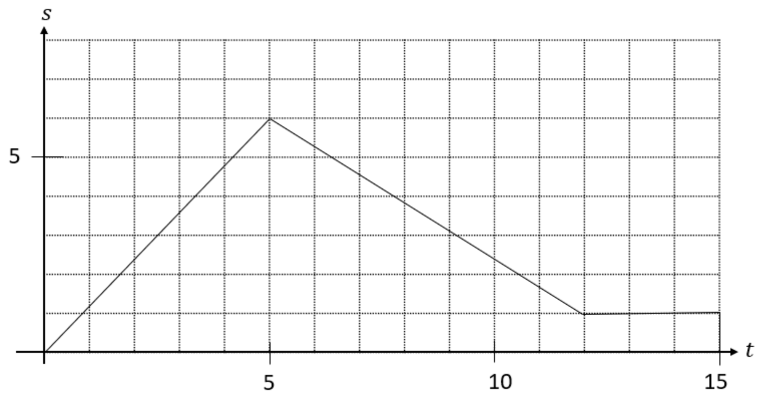
2. Graphs of rectilinear motion

(a) Displacement-time graph



Gradient of graph: velocity, ms^{-1}
Area under graph: -

Example:

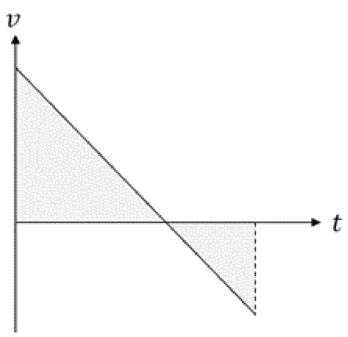


Given the displacement time graph above, find the velocity function for time $5 \leq t \leq 12$.
Hence, find the acceleration from $t = 6\ s$ to $t = 10\ s$.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{6-1}{5-12}(x - 12)$$
$$y = -\frac{5}{7}x + 9\frac{4}{7}$$
$$v(t) = -\frac{5}{7}t + 9\frac{4}{7} \quad for\ 5 \leq t \leq 12$$

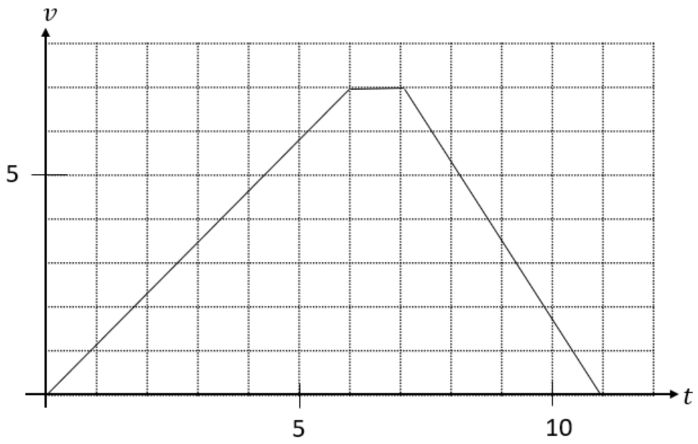
$$a(t) = \int_{6}^{10} -\frac{5}{7}t + 9\frac{4}{7}dt$$
$$= \left[\frac{36x}{7} - \frac{5x^2}{14}\right]_6^{10}$$
$$= \frac{110}{7} - 18$$
$$= -\frac{16}{7}ms^{-2}$$

(b) Velocity-time graph



Gradient of graph: acceleration, ms^{-2}
Area under graph: displacement, m

Example:



Given the velocity-time graph of a moving particle above, what is the time when particle was 28 m away from its' origin?

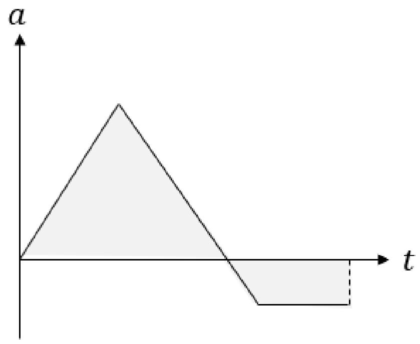
Distance travelled for $0 \leq t \leq 6$,
$\frac{1}{2} \times 6 \times 7 = 21\ m$

Distance left $= 28 - 21$
$= 7\ m$

From $6 \leq t \leq 7$, v is constant at $7\ ms^{-1}$
$v = \frac{d}{t}$
$7 = \frac{7}{t}$
$t = 1\ s$

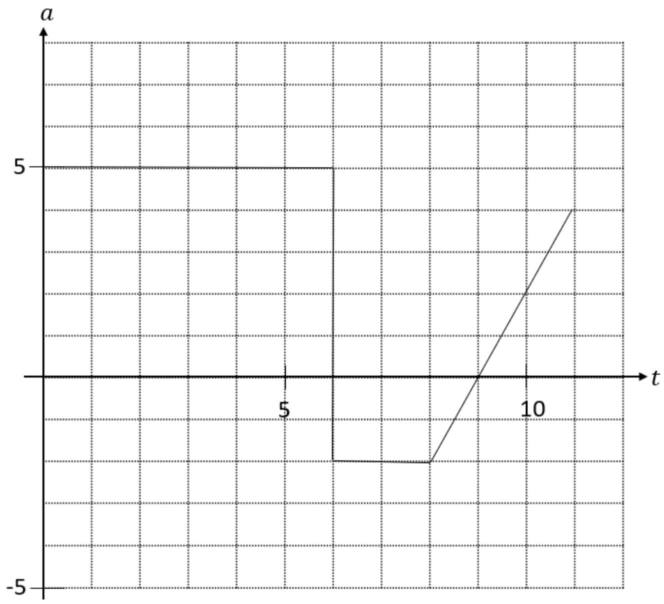
Therefore, time when particle was 28 m away from its' origin is $6 + 1 = 7\ s$.

(c) Acceleration-time graph



Gradient of graph: jerk, ms^{-3}
Area under graph: velocity, v

Example:



A person observes the motion of the particle at the 9[th] s of the particle's journey from its origin. Find the velocity of the particle after 2 s from the time when the person observes the motion of particle.

Velocity = area under the graph

$$v = \frac{1}{2}[(9+2)-9](4)$$
$$= 4\ ms^{-1}$$

END